A REMARK ON EXTREMAL FUNCTION FOR TRUDINGER-MOSER INEQUALITY WITH REMAINDER TERMS

Yu Fang

Abstract

Let Ω be a smooth bounded domain in $\mathbb{R}^2$, $h : \Omega \to \mathbb{R}$ be a function such that $-\Delta + h$ is coercive, where Δ is the Laplacian operator. We prove the existence of extremal function for the following Trudinger-Moser inequality

$$\sup_{\int_\Omega (|\nabla u|^2 + hu^2)dx \leq 1} \int_\Omega e^{4\pi u^2} dx < \infty.$$

Such kind of inequalities was originally proposed by Adimurthi-Druet [Adimurthi and O. Druet, Blow-up analysis in dimension 2 and a sharp form of Trudinger-Moser inequality, Comm. Partial Differential Equations 29(1-2) (2004), 295-322], generalized by Tintarev [C. Tintarev, Trudinger-Moser inequality with remainder terms, J. Funct. Anal. 266 (2014), 55-66] and Yang [Y. Yang, Extremal functions for Trudinger-Moser inequalities of Adimurthi-Druet type in dimension two, J. Differential Equations 258 (2015), 3161-3193]. This generalizes part of the known results. Our method is the usual blow-up analysis.